International Journal of Mathematics and
Computer Applications Research (IJMCAR)
ISSN(P): 2249-6955; ISSN(E): 2249-8060
Vol. 4, Issue 4, Aug 2014, 33-46

NUMERICAL BEHAVIOR OF INSTABILITY IN DISPLACEMENT PROCESS THROUGH HOMOGENEOUS POROUS MEDIA

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ABSTRACT

In this paper, we have numerically discussed the phenomenon of instabilities in a displacement process involving two immiscible liquids. The phenomenon is considered without involving the magnetic fluid. Numerical solution of governing non-linear partial differential equation for the phenomenon has been obtained by Finite element techniques. Finite element technique is a numerical method for finding an approximate solution of differential equation in finite region or domain. A Matlab code is developed to solve the problem and the numerical results are obtained at various time levels.

KEYWORDS: Homogeneous Porous Media, Fluid flow, Finite Element Technique

INTRODUCTION

If a fluid contained in a porous medium is displaced by another fluid of lesser viscosity, then it is frequently observed that the displacing fluid has a strong tendency to protrude in form of fingers (instabilities) into more viscous fluid. This phenomenon is called fingering.

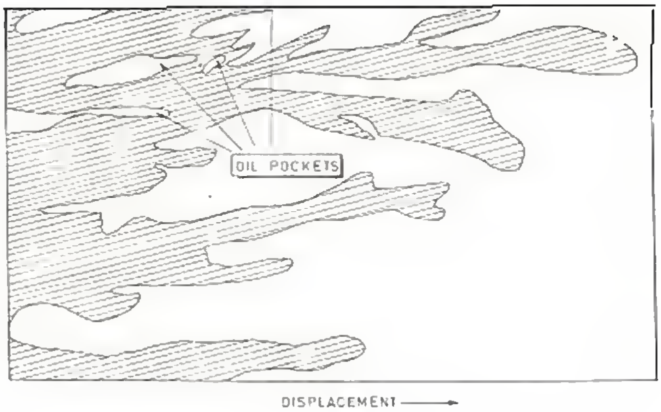

Figure 1: Fingering Process in a Porous Medium Oil Occupies Unshaded Area and Water Occupies Shades Area

In Figure 1, fingering process has been shown between oil-water flows into a porous medium. In petroleum engineering, the fingering process is a well known phenomenon occurring in displacement of oil by water flooding that is a common oil recovery technique.

In the statistical treatment of fingering [1] only average cross-sectional area occupied by the fingers was observed while the size and shape of the individual fingers are neglected as in figure 2. Scheidegger and Johnson [21] discussed the statistical behavior in homogeneous porous media with capillary pressure. Verma [22] has examined the behavior of fingering in a displacement process through heterogeneous porous media.

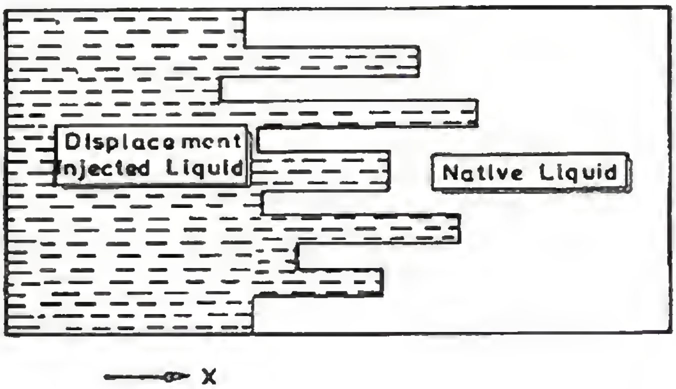

Figure 2: Schematic Representation of Fingers at Level 'x'

In the present paper, we have obtained a numerical solution of the problem by finite element techniques using Matlab.

STATEMENT OF THE PROBLEM

We consider that there is a uniform water injection into an oil saturated porous medium of homogeneous physical characteristics, such that the injecting water cuts through the oil formation and give rise to protuberance. This furnishes a well-developed fingers flow. Since the entire oil at the initial boundary (x=0) is displaced through a small distance due to the water injection. Therefore, we assume, further that complete water saturation exists at the initial boundary.

MATHEMATICAL FORMULATION OF THE PROBLEM

The seepage velocity of water (V_w) and oil (V_o) are given by Darcy's Law,

$$V_w = -\left(\frac{K_w}{\delta_w}\right)K\left[\frac{\partial P_w}{\partial x}\right] \tag{1}$$

$$V_o = -\left(\frac{K_o}{\delta_o}\right)K\left[\frac{\partial P_o}{\partial x}\right] \tag{2}$$

Where K is the permeability of the homogeneous medium, K_w and K_o are relative permeability of water and oil, which are functions of S_w and S_o (S_w and S_o are the saturation of water and oil) respectively, P_w and P_o are pressure of water and oil, δ_w and δ_o are constant kinematics viscosities, α is the inclination of the bed and g is acceleration due to gravity.

Regarding the phase densities as constant, the equations of continuity of the two phases are:

$$P\left(\frac{\partial S_w}{\partial t}\right) + \left(\frac{\partial V_w}{\partial x}\right) = 0 \tag{3}$$

$$P\left(\frac{\partial S_o}{\partial t}\right)+\left(\frac{\partial V_o}{\partial x}\right)=0 \tag{4}$$

Where, P is porosity of the medium. From the definition of phase saturation, it is evident that, $S_w + S_o = 1$ (5)

The capillary pressure P_c is defined as

$$P_c = -\beta_o S_w \tag{6}$$

$$P_c = P_o - P_w \tag{7}$$

Where, β_o is a constant quantity.

At this state, for definiteness of the mathematical analysis, we assume standard relationship due to Scheidegger and Johnson [21], between phase saturation and relative permeability as

$$K_w = S_w \tag{8}$$

$$K_o = \quad S_0 = 1 - S_w \tag{9}$$

The equation of motion for saturation can be obtained by substituting the values of V_w and V_o from equations (1) and (2) into the equations (3) and (4) respectively, we get,

$$P\left(\frac{\partial S_w}{\partial t}\right)=\frac{\partial}{\partial x}\left[\left(\frac{K_w}{\delta_w}\right)K\left[\frac{\partial P_w}{\partial x}\right]\right] \tag{10}$$

$$P\left(\frac{\partial S_o}{\partial t}\right)=\frac{\partial}{\partial x}\left[\left(\frac{K_o}{\delta_o}\right)K\left[\frac{\partial P_o}{\partial x}\right]\right] \tag{11}$$

These are the general flow equation of the phase in homogeneous medium, when effects due to pressure discontinuity and gravity term in inclined porous medium are considered.

Eliminating $\left(\frac{\partial P_w}{\partial x}\right)$ from equations (10) and (7), we obtain

$$P\left(\frac{\partial S_w}{\partial t}\right)=\frac{\partial}{\partial x}\left[\left(\frac{K_w}{\delta_w}\right)K\left(\frac{\partial P_o}{\partial x}-\frac{\partial P_C}{\partial x}\right)\right] \tag{12}$$

Combining equation (11) and (12) and using equation (5), we get

$$\frac{\partial}{\partial x}\left[K\left(\frac{K_w}{\delta_w}+\frac{K_o}{\delta_o}\right)\frac{\partial P_o}{\partial x}-K\frac{K_w}{\delta_w}\frac{\partial P_C}{\partial x}\right]=0$$

Integrating above eq. with respect to x, we have

$$K\left(\frac{K_w}{\delta_w}+\frac{K_o}{\delta_o}\right)\frac{\partial P_o}{\partial x}-K\frac{K_w}{\delta_w}\frac{\partial P_C}{\partial x} = -V \tag{13}$$

Where, V is constant of integrating which can be evaluated later on. Simplification of (13) gives,

$$\frac{\partial P_o}{\partial x}=\frac{-V}{K\left(\frac{K_w}{\delta_w}+\frac{K_o}{\delta_o}\right)}+\frac{\left(\frac{\partial P_C}{\partial x}\right)}{1+\left(\frac{K_o}{K_w}\right)\left(\frac{\delta_w}{\delta_o}\right)}$$

Using above equation in equation (12), we have

$$P\frac{\partial S_w}{\partial t}+\frac{\partial}{\partial x}\left[\frac{V}{K\left(\frac{K_w}{\delta_w}+\frac{K_o}{\delta_o}\right)}+\frac{\left(\frac{K_o}{\delta_o}\right)\left(\frac{\partial P_C}{\partial x}\right)}{1+\left(\frac{K_o}{K_w}\right)\left(\frac{\delta_w}{\delta_o}\right)}\right]=0 \tag{14}$$

The value of pressure of oil (P_o) can be written as in [23] in the form

$$P_o=\frac{P_o+P_w}{2}+\frac{P_o-P_w}{2}=\overline{P}+\frac{1}{2}P_C \tag{15}$$

Where, $\overline{P}$ is the mean pressure which is constant, therefore (15) implies $\frac{\partial P_o}{\partial x}=\frac{1}{2}\frac{\partial P_C}{\partial x}$

Using above equation in (13) we get,

$$V=\frac{K}{2}\left[\left(\frac{K_w}{\delta_w}\right)-\left(\frac{K_o}{\delta_o}\right)\frac{\partial P_C}{\partial x}\right]$$

Substituting the value of V from above equation in equation (14) we get,

$$P\frac{\partial S_w}{\partial t}+\frac{1}{2}\frac{\partial}{\partial x}\left[K\left(\frac{K_w}{\delta_w}\right)\left(\frac{dP_C}{dS_w}\right)\left(\frac{\partial S_w}{\partial x}\right)\right]=0$$

Substituting the value of K_w and P_C from (6) and (8) we get,

$$P\frac{\partial S_w}{\partial t}-\frac{\beta_o K}{2\delta_w}\frac{\partial}{\partial x}\left[S_w\frac{\partial S_w}{\partial x}\right]=0 \tag{16}$$

A set of suitable boundary conditions associated to problem (16) are

$S_w(0,t) = 1$; $S_w(x,0) = 0$; $S_w(L,t) = 0$ (17)

Equation (16) is reduced to dimensionless form by setting

$$X=x/L,\ T=\frac{K\beta_o t}{2\delta_w L^2 P},\ S_w(x,t)=S_w^*(X,T)$$

So that $$\frac{\partial S_w}{\partial T} = \frac{\partial}{\partial X}\left(S_w \frac{\partial S_w}{\partial X}\right) \tag{18}$$

With auxiliary conditions

$$S_w(0,T) = 1 \; ; \; S_w(X,0) = 0 \; ; \; S_w(1,T) = 0 \tag{19}$$

In equation (18) and (19) the asterisk are dropped for simplicity.

Equation (18) is desired nonlinear differential equation of motion for the flow of two immiscible liquids in homogeneous medium.

A Matlab Code is prepared and executed with **h =1/15** and **k = 0.002223** for **225** time levels. The numerical value are shown by table. Curves indicating the behavior of saturation of water corresponding to various time periods.

FINITE ELEMENT METHOD

We attempt to solve the time dependent one-dimensional problem (18) by the application of finite element technique. We discuss the details of semi-discrete variational formulation of the problem.

The domain of the problem, in present case, consists of all points between $x = 0$ and $x = 1$ Figure 3(a). This domain is divided into set of linear elements Figure 3(b).

Figure 3(a)

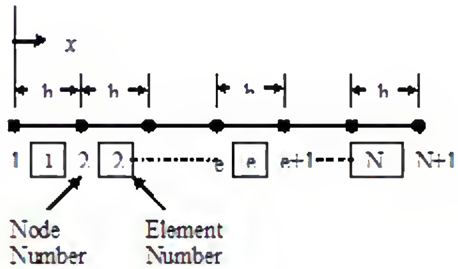

Figure 3(b)

Now, the variational form of given PDE (18) is

$$J(S_w) = \frac{1}{2}\int_R \left[S_w \left(\frac{\partial S_w}{\partial X}\right)^2 + 2S_w \frac{\partial S_w}{\partial T} \right] dX \tag{20}$$

Choose an arbitrary linear element $R^{(e)} = [S_1^{(e)}, S_2^{(e)}]$ & obtain interpolation function for $R^{(e)}$ using Lagrange interpolation Method such as

$$S^{(e)}(X) = \sum_{j=1}^{2} N_j(X) S_j^{(e)} = N^{(e)}\phi^{(e)} = \phi^{(e)T} N^{(e)T} \tag{21}$$

$$where \qquad N^{(e)} = [N_1 \; N_2] \quad \& \quad \phi^{(e)} = [S_1 \quad S_2]^T$$

For linear element, Shape function is

$$N_1 = \frac{X - X_1}{X_2 - X_1} \quad and \quad N_2 = \frac{X_2 - X}{X_2 - X_1}$$
$$\ni \quad N_1(X) + N_2(X) = 1$$
$$N_j(X_i) = \begin{cases} 1 & j = i \\ 0 & j \neq i \end{cases} \tag{22}$$

Now, apply Variational Method to $R^{(e)}$,therefore equation (20) becomes

$$J\left(S^{(e)}\right) = \frac{1}{2} \int\limits_{R^{(e)}} \left[S^{(e)} \left(\frac{\partial S^{(e)}}{\partial X} \right)^2 + 2S^{(e)} \frac{\partial S^{(e)}}{\partial T} \right] dX \tag{23}$$

$$as, \quad S^{(e)}(X) = N^{(e)} \phi^{(e)} = \phi^{(e)^T} N^{(e)^T}$$
$$\therefore \quad \frac{\partial S^{(e)}}{\partial X} = \frac{\partial N^{(e)}}{\partial X} \phi^{(e)} = \phi^{(e)^T} \frac{\partial N^{(e)^T}}{\partial X} \, and \left(\frac{\partial S^{(e)}}{\partial X} \right)^2 = \phi^{(e)^T} \frac{\partial N^{(e)^T}}{\partial X} \frac{\partial N^{(e)}}{\partial X} \phi^{(e)}$$

Therefore equation (23) becomes,

$$J\left(S^{(e)}\right) = \frac{1}{2} \int\limits_{R^{(e)}} \phi^{(e)^T} \left[N^{(e)} \phi^{(e)} \left(\frac{\partial N^{(e)^T}}{\partial X} \frac{\partial N^{(e)}}{\partial X} \right) \phi^{(e)} + 2\left(N^{(e)^T} N^{(e)} \right) \frac{\partial \phi^{(e)}}{\partial T} \right] dX \tag{24}$$

For minimization, first differentiate equation (24) with respect to $\phi^{(e)}$

$$\frac{\partial J^{(e)}}{\partial \phi^{(e)}} = \int\limits_{R^{(e)}} \left[N^{(e)} \phi^{(e)} \left(\frac{\partial N^{(e)^T}}{\partial X} \frac{\partial N^{(e)}}{\partial X} \right) \phi^{(e)} + 2\left(N^{(e)^T} N^{(e)} \right) \frac{\partial \phi^{(e)}}{\partial T} \right] dX$$

Now, $\dfrac{\partial J^{(e)}}{\partial \phi^{(e)}} = 0$

$\therefore$ The Element equation is $A^{(e)} \dfrac{\partial \phi^{(e)}}{\partial T} + B^{(e)}(\phi^{(e)}) \phi^{(e)} = 0$ (25)

Where $A^{(e)} = \displaystyle\int_{S_1^{(e)}}^{S_2^{(e)}} \left(N^{(e)^T} N^{(e)} \right) dX$, $\qquad B^{(e)}(\phi^{(e)}) = \displaystyle\int_{S_1^{(e)}}^{S_2^{(e)}} N^{(e)} \phi^{(e)} \left(\frac{\partial N^{(e)^T}}{\partial X} \frac{\partial N^{(e)}}{\partial X} \right) dX$

We use Gauss Legendre Quadrature Method to evaluate these integral. For this, first we transform co-ordinate X to a local coordinate z $\ni$ when $X = S_1$, $z = -1$; when $X = S_2$, $z = 1$.

Therefore Interpolation function becomes $N_1(z)=\frac{1}{2}(1-z)$; $N_2(z)=\frac{1}{2}(1+z)$ and Jacobian matrix is $J=\frac{1}{2}h$. Also element matrix transform to

$$A^{(e)}=\int_{-1}^{1}\left(N^{(e)T}N^{(e)}\right)Jdz\approx\sum_{I=1}^{r}A^{(e)}(z_I)W_I$$

$$B^{(e)}(\phi^{(e)})=\int_{-1}^{1}N^{(e)}\phi^{(e)}\left(\frac{1}{J}\frac{\partial N^{(e)T}}{\partial X}\frac{1}{J}\frac{\partial N^{(e)}}{\partial X}\right)Jdz\approx\sum_{I=1}^{r}B^{(e)}(z_I)W$$

For $A^{(e)}$, degree of polynomial p =2 then r = 2 and For $B^{(e)}$, p = 1 then r =1. z_I and W_I are corresponding gauss points and gauss weights with respect to 'r'. Then, the element matrix becomes,

$$A^{(e)}=\frac{h^{(e)}}{6}\begin{bmatrix}2 & 1\\ 1 & 2\end{bmatrix},\quad B^{(e)}(\phi^{(e)})=\frac{1}{2h^{(e)}}\begin{bmatrix}S_1+S_2 & -S_1-S_2\\ -S_1-S_2 & S_1+S_2\end{bmatrix} \tag{26}$$

ASSEMBLING OF ELEMENTS

In deriving the element equations, we isolated a typical element (the eth element) from the mesh and formulated the variational problem and developed its finite element model. To obtain the finite element equations of the total problem, we must put the elements back into their original positions. The assembly of linear elements is carried out by imposing the following two conditions:

- The continuity of primary variable requires

$$S_n^e=S_1^{e+1}$$

- The balance of secondary variables at connecting nodes requires

$$S_1^1=S_1,\quad S_2^1=S_1^2=S_2,\quad S_2^2=S_1^3=S_3,\ldots\ldots,S_2^{N-1}=S_1^N=S_N,\quad S_2^N=S_{N+1}$$

$$S_1^{e+1}=\begin{cases}0 & \text{if no external point surce is applied}\\ S_1 & \text{if an external point surce of magnitude} S_1 \text{ is applied}\end{cases}$$

The inter-element continuity of primary variable can be imposed by simply renaming the variables of all elements connected to common node. For example, for a mesh of N linear finite element connected in series, we have

For a uniform mesh of N elements, by equation (25) & equation (26), the assembled equation becomes,

$$A\frac{\partial\varphi}{\partial T}-B(\phi)\phi=0 \tag{27}$$

where, $$A = \frac{h}{6}\begin{bmatrix} 2 & 1 & 0 & 0 & \cdots & 0 & 0 \\ 1 & (2+2) & 1 & 0 & \cdots & 0 & 0 \\ 0 & 1 & (2+2) & 0 & \cdots & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & 0 & \cdots & (2+2) & 1 \\ 0 & 0 & 0 & 0 & \cdots & 1 & 2 \end{bmatrix}$$

$$B(\phi) = \frac{1}{2h}\begin{bmatrix} S_1+S_2 & -S_1-S_2 & 0 & 0 & \cdots & 0 & 0 \\ -S_1-S_2 & S_1+2S_2+S_3 & -S_2-S_3 & 0 & \cdots & 0 & 0 \\ 0 & -S_2-S_3 & S_2+2S_3+S_4 & -S_3-S_4 & \cdots & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & 0 & \cdots & S_{14}+2S_{15}+S_{16} & -S_{15}-S_{16} \\ 0 & 0 & 0 & 0 & \cdots & -S_{15}-S_{16} & S_{15}+S_{16} \end{bmatrix}$$

$$\phi = [S_1 \quad S_2 \quad \cdots \quad \cdots \quad S_{16}]^T \tag{28}$$

Equation (27) represents the assembled equation.

TIME APPROXIMATION

We have obtained the finite element equation in the global form, which represent a system of simultaneous ordinary differential equation. We now introduced δ family of approximations which approximates weighted average of a dependent variable of two consecutive time steps by linear interpolation of the values of the variable at the two time steps such as $S_j = \delta S_j^{n+1} + (1-\delta)S_j^n$ (29)

The time derivatives $\dot{\theta}_j$ are replaced by Forward finite difference formula such as, $\dot{S}_j = \frac{S_j^{n+1} - S_j^n}{k}$ (30)

In view of (29) and (30), equation (27) written as,

$[A + \delta\, k(B(\phi^{(n+1)}))\,]\phi^{(n+1)} = [\, A - (1-\delta)\, k(B(\phi^{(n)}))\,]\phi^{(n)}$

Where **δ =1/2** and n = 0,1,2,……

For a uniform mesh of N elements, by equation (28), the above equation takes the form,

$$\left[K(\phi^{(n+1)})\right]\phi^{(n+1)} = \left[F_1(\phi^{(n)})\right]\phi^{(n)} = F(\phi^{(n)}) \tag{31}$$

Equation (31) represents the global form of the problem.

IMPOSING BOUNDARY CONDITIONS

The boundary condition $S_w(0, T) = 1$ states that $S_1^{(n+1)} = 1$ for all $n \geq 0$. Therefore we replace all the entries of 1st row of matrix K by zero except the diagonal entry K_{11} replaced by one and replace 1st row of matrix F by 1 and boundary

condition $S_w(L, T) = 0$ states that $S_{N+1}^{(n+1)} = 0$ for all $n \geq 0$. Therefore we replace all the entries of $(N+1)^{th}$ row of matrix K by zero except the diagonal entry K_{NN} replaced by one and replace $N+1)^{th}$ row of matrix F by 0. Thus, by applying boundary condition (19) to equation (31) and simplifying, we get

$$\left[K\phi^{(n+1)}\right]\phi^{(n+1)} = F \tag{32}$$

Where

$$\left[K(\phi^{(n+1)}\right] =$$

$$\frac{1}{2h}\begin{bmatrix} 1 & 0 & 0 & 0 & \cdots & 0 & 0 \\ \frac{h}{6}-\frac{\delta k}{2h}\left(S_1^{(n+1)}+S_2^{(n+1)}\right) & \frac{2h}{3}+\frac{\delta k}{2h}\left(S_1^{(n+1)}+2S_2^{(n+1)}+S_3^{(n+1)}\right) & \frac{h}{6}-\frac{\delta k}{2h}\left(S_2^{(n+1)}+S_3^{(n+1)}\right) & 0 & \cdots & 0 & 0 \\ 0 & \frac{h}{6}-\frac{\delta k}{2h}\left(S_2^{(n+1)}+S_3^{(n+1)}\right) & \frac{2h}{3}+\frac{\delta k}{2h}\left(S_2^{(n+1)}+2S_3^{(n+1)}+S_4^{(n+1)}\right) & \frac{h}{6}-\frac{\delta k}{2h}\left(S_3^{(n+1)}+S_4^{(n+1)}\right) & \cdots & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & 0 & \cdots & \frac{2h}{3}+\frac{\delta k}{2h}\left(S_{14}^{(n+1)}+2S_{15}^{(n+1)}+S_{16}^{(n+1)}\right) & \frac{h}{6}-\frac{\delta k}{2h}\left(S_{15}^{(n+1)}+S_{16}^{(n+1)}\right) \\ 0 & 0 & 0 & 0 & \cdots & 0 & 1 \end{bmatrix}$$

$$\left[K(\phi^{(n+1)}\right] = \phi^{(n+1)} = \begin{bmatrix} S_1^{(n+1)} \\ S_2^{(n+1)} \\ S_3^{(n+1)} \\ \vdots \\ \vdots \\ S_{15}^{(n+1)} \\ S_{16}^{(n+1)} \end{bmatrix}$$

F=

$$\begin{bmatrix} 1 \\ \frac{h}{6}+(1-\delta)k\left(S_1^{(n)}+S_2^{(n)}\right)S_1^{(n)}+\frac{2h}{3}-(1-\delta)k\left(S_1^{(n)}+2S_2^{(n)}+S_3^{(n)}\right)S_2^{(n)}+\frac{h}{6}-(1-\delta)k\left(S_2^{(n)}+S_3^{(n)}\right)S_3^{(n)} \\ \vdots \\ \frac{2h}{3}-(1-\delta)k\left(S_{14}^{(n)}+2S_{15}^{(n)}+S_{16}^{(n)}\right)S_{15}^{(n)}+\frac{h}{6}-(1-\delta)k\left(S_{15}^{(n)}+S_{16}^{(n)}\right)S_{16}^{(n)} \\ 0 \end{bmatrix}$$

Thus, equation (32) is the resulting system of non linear algebraic equation.

SOLUTION OF NON-ALGEBRAIC EQUATION

In the previous section, we obtained the assembled equation which is nonlinear. The assembled nonlinear equations after imposing boundary conditions are given by equation (32). We seek an approximate solution by the

linearization which based on scheme $\left[K\phi^{(n)}\right]\phi^{(n+1)} = F$ (33)

Where, $\phi^{(n)}$ denotes the solution of the n iteration. Thus, the coefficient K_{ij} are evaluated using the solution $\phi^{(n)}$ from the previous iteration and the solution at the $(n+1)^{th}$ iteration can be obtained by solving equation (33) using Gauss Elimination Method. At the beginning of the iteration (i.e. n=0), we assume the solution $\phi^{(0)}$ from initial condition which requires to have $S_1^{(0)} = S_2^{(0)} = \ldots\ldots\ldots\ldots\ldots = S_{N+1}^{(0)} = 0$.

Graphical Representation

A Matlab Code is prepared for 15 elements model and resulting equation (32) for N = 15 is solved by Gauss Elimination method.

Saturation of injected liquid at time t = 0.1, t = 0.2, t = 0.3, t = 0.4 and t = 0.5 seconds are

Table 1

1.0000e+000	1.0000e+000	1.0000e+000	1.0000e+000	1.0000e+000
9.0224e-001	9.3166e-001	9.4451e-001	9.5211e-001	9.5727e-001
7.9536e-001	8.5880e-001	8.8602e-001	9.0198e-001	9.1275e-001
6.7900e-001	7.8129e-001	8.2447e-001	8.4956e-001	8.6641e-001
5.5278e-001	6.9899e-001	7.5977e-001	7.9481e-001	8.1821e-001
4.1595e-001	6.1178e-001	6.9187e-001	7.3769e-001	7.6813e-001
2.6577e-001	5.1952e-001	6.2069e-001	6.7814e-001	7.1611e-001
9.7552e-002	4.2201e-001	5.4613e-001	6.1612e-001	6.6211e-001
1.9660e-002	3.1880e-001	4.6813e-001	5.5159e-001	6.0607e-001
5.6084e-003	2.0889e-001	3.8656e-001	4.8449e-001	5.4786e-001
1.4765e-003	9.4375e-002	3.0138e-001	4.1474e-001	4.8728e-001
3.9746e-004	1.3564e-002	2.1285e-001	3.4228e-001	4.2388e-001
1.0633e-004	4.0701e-003	1.2138e-001	2.6710e-001	3.5668e-001
2.8364e-005	1.0496e-003	1.2693e-002	1.8965e-001	2.8319e-001
7.0903e-006	2.6478e-004	2.6724e-003	1.1219e-001	1.9564e-001
0	0	0	0	0

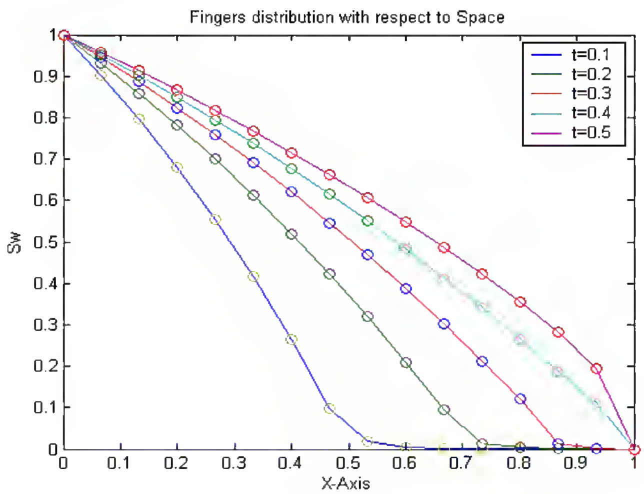

Figure 4

Interpretation

In all graphs, X-axis represents the saturation of injected liquid (S_w) of porous media of length one. Y-axis represents the time 't' in seconds. Solution obtained with **h =1/15** and **k = 0.002223** for **225** time levels.

Initially, saturation of injected liquid is zero at each point on observed region. Also, there is full injected liquid saturation (i.e. $S_w = 1$) at injected face x = 0 & there is no saturation of injected liquid at other end (x =1) irrespective of time.

It is clear from graph that, for each value of T, Saturation Sw has a decreasing tendency along the space co-ordinate axis. Also, for each point of X, the Saturation increases as time increases but the rate at which it rises at each point in observed region slows down with increase in time. This shows that the stabilization of the fingers is truly possible with the assumptions made for capillary pressure and water saturation.

CONCLUSIONS

Engineers in several fields have to learn the mechanism of drainage and to apply to problems of water supply, land reclamation and stabilization of foundations and sub grade, and also to the fields of petroleum production and agriculture.

Drainage in general is any provision for the removal of excess water. The common objective of land projects to prevent or eliminate either water logging or inundation or otherwise productive land. Drainage of projected land refers principally to the disposal of surplus natural water adversely affecting irrigation. Practically every area where irrigation has been carried on for time has been affected by high water table. Therefore provision for adequate drainage is an essential part of planning, construction and operation of an irrigation project.

For agriculture purpose, the continued presence of water in excess of that needed for vegetation is harmful. Prolonged saturation of soil excludes air essentially for healthy plant growth and the soil becomes cold, sour and unproductive. Consequently unsaturated or irrigated soils is a necessary evil, so to this type of drainage where originally saturation conditions are existing up to the top.

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